(A_0 is initial amplitude). Whence it follows that low-frequency and high-frequency losses lead to different rules for the change in wave amplitude. For the example given, an estimate of coefficients in Eq. (12) indicates that $\zeta \sim 10^{-12}(\Lambda/a)$ and $14\,\alpha_H\delta/3\beta \sim 2\cdot10^{-14}(\Lambda/a)$. Consequently, with strains $A \leqslant 1(u_x \leqslant 10^{-5})$ high-frequency losses are negligibly small, and therefore it is possible to ignore the second term in the denominator of (13), and as a result of this the soliton amplitude will also decrease by an exponential rule but with another decrement.

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ANALYSIS OF THE POWDER COMPACTION PROCESS IN A CYLINDRICAL CONTAINER ON THE BASIS OF A SIMPLE MODEL

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Explosive compaction of powders is often accomplished in cylindrical geometry when the applied load is quite large and, as a result of this loading, may affect the strength properties of materials. A similar point of view was expressed in [1], and this is also indicated by experimental results [2-4]. During shock loading the final powder density, shock wave (SW) amplitude, and the strength properties of the compacted material appear to be connected with each other in a complex fashion. However, since the main change in powder volume occurs in the shock-wave front (SWF), as a first approximation the change in density behind the front is ignored, and it is assumed to be constant. In addition, there is one more severe simplification, i.e., the dynamic yield strength is assumed to be constant. With detonation rates much greater than the SW velocity in the powder, the slope of it to the container axis is small, and for analysis it is possible to use a unidimensional model.

In a unidimensional arrangement the problem of loading a compacting cylinder without a shell was resolved in [5]; the case was studied numerically for constant load and dynamic yield strength depending linearly on internal energy of the material, and also the asymptotic behavior was found for SW amplitude at the start and end of the process of its convergence.

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In [6, 7] calculation was carried out for powder compaction by powerful explosive in a cylindrical container filled with a two-dimensional non-steady-state model. An equation for the state of the powder was worked out according to a model in [8]. Results were given for calculation in specific experimental arrangements of normal stresses in the SWF for different thicknesses of explosive layer and also the field of velocities with loading of powders with different dynamic yield strengths.

In practical applications a knowledge is normally required of the main parameters governing the powder compaction process in cylindrical containers and the degree of their effect on this process. In this connection, in experimental works there exist different points of view, and correspondingly different approaches, to determining the optimum compaction regime. In [4] governing parameters are assumed to be the ratio of explosive mass to that of the powder and the square of the detonation velocity, in [2] they are assumed to be density, shell strength, shell thickness, and explosive layer thickness, in [9] they are assumed to be the ratio of the explosive mass to that of the container with the powder, and in [3] the energy which is transferred to the powder by the container, accelerated detonation products, and it depends on geometric dimensions of the shell and its hardness, and also on the ratio of explosive mass to that of the shell; none of them are universal [9].

To a considerable extent an approach applied in [5] is used in the present work. On the basis of a simple model, the problem is resolved for powder compaction in a cylindrical container for which it is possible to obtain an idea about the effect of different initial parameters on SW behavior in the powder.

1. We consider loading by external pressure p of material (powder) with initial density ρ_0 in an incompressible cylindrical shell with density ρ_c with constant dynamic yield strength σ_c and radii a_0 (inner) and b_0 (outer). The stressed state is assumed to be plane-strained. Ahead of the SWF the material does not exhibit strength. With any SW amplitude behind the SWF in the powder, a condition is attained with density ρ_p and dynamic yield strength σ_p having a constant value.

Equations of motion and incompressibility in cylindrical coordinates in the case of axial symmetry are

$$\rho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}\right)=\frac{\partial\sigma_r}{\partial r}-\frac{\sigma_r-\sigma_\varphi}{r}; \tag{1.1}$$

$$\frac{\partial(vr)}{\partial r}=0. \tag{1.2}$$

Here σ_r and σ_φ are stress tensor components in a cylindrical coordinate system; ρ and t are density and time; v and r are mass velocity and distance to the container axis (Euler variables). Mises flow conditions are used $\sigma_r-\sigma_\varphi=\sigma_i$ (σ_i is minimum yield strength of the container material or powder). It follows from (1.2) that

$$vr=f(t)=aa'=bb'=\lambda xx', \tag{1.3}$$

where $\lambda = 1 - \rho_0/\rho_p$ is powder porosity; $a=(a_0^2-\lambda b_0^2+\lambda x^2)^{1/2}$, $b=(b_0^2-\lambda b_0^2+\lambda x^2)^{1/2}$ are current outer and inner shell radii; x = x(t) is SW radius in the powder; a prime indicates differentiation with respect to time.

By using (1.3), from (1.1) we obtain

$$\rho(f'/r-f^2/r^3)=\partial\sigma_r/\partial r-\sigma_i/r; \tag{1.4}$$

$$f'=\lambda xx''+\lambda(x')^2. \tag{1.5}$$

We integrate (1.4) with respect to r once from a to x, and a second time from r(x ≤ r ≤ b) to x, and we use (1.5), boundary conditions $\sigma_r(x) = -\rho_0\lambda(x')^2$ and $\sigma_r(a) = -p(t)$, and continuity condition σ_r with r = b. As a result of this

$$x''=\frac{\rho_0\lambda(x')^2(1-R-K)+S-p}{\rho_0\lambda xR}, \tag{1.6}$$

$$-\sigma_r(r,t)=\rho_0\lambda(x')^2-\rho_p\lambda[xx''+(x')^2]\ln\frac{r}{x}+\sigma_p\ln\frac{r}{x}+\frac{\rho_p\lambda(x')^2}{2}\left(1-\frac{x^2}{r^2}\right),$$

$$R=\frac{\rho_c}{\rho_0}\ln\frac{a}{b}+\frac{\rho_p}{\rho_0}\ln\frac{b}{x},\quad K=\frac{\lambda x^2}{2}\left[\frac{\rho_c}{\rho_0}\left(\frac{1}{a^2}-\frac{1}{b^2}\right)+\frac{\rho_p}{\rho_0}\left(\frac{1}{b^2}-\frac{1}{x^2}\right)\right],$$

$$S=\sigma_c\ln\frac{a}{b}+\sigma_p\ln\frac{b}{x}.$$

In this model work A provided by external pressure is consumed in communicating kinetic energy E_c to the substance, in the increase in internal energy of the substance as a result of plastic deformation at the front E_1, and behind the SWF E_2. For these values per unit length of container, we have

$$A(x) = -2\pi \int_{a_0}^{a} pada = -2\pi\rho\lambda \int_{b_0}^{x} pxdx, \tag{1.7}$$

$$Ec(x) = -2\pi \int_{a}^{x} \frac{\rho v^2}{2} rdr = \pi\rho_0\lambda^2 x^2 (x')^2 R,\ E_1(x) = -\pi\rho_0\lambda^2 \int_{b_0}^{x} (x')^2 xdx.$$

In order to obtain relationship (1.7), we use (1.3) and an expression for the increase in internal energy in the SWF per unit mass $\Delta E_1 = \lambda^2(x')^2/2$. The work of plastic deformation of the substance behind the SWF is written in the form

$$E_2(x) = -\pi\sigma_c\left(a^2 \ln\frac{a}{b_0} - a_0^2 \ln\frac{a_0}{b_0} - b^2 \ln\frac{b}{b_0}\right) - \pi\sigma_p\left(b^2 \ln\frac{b}{b_0} - \lambda x^2 \ln\frac{x}{b_0}\right). \tag{1.8}$$

In any instant of time $A = E_c + E_1 + E_2$; the correctness of this equality may be checked by differentiating it with respect to time using (1.6)-(1.8).

In (1.6)-(1.8) we transfer to dimensionless variables by the equations

$$r = b_0 r^*,\ \rho_i = \rho_0\rho_i^*,\ \sigma_i = p_0\sigma_i^*,\ p = p_0 p^*,$$
$$a_0 = b_0 a_0^*,\ x = b_0 x^*,\ t = \tau t^*,\ E_i = CE_i^*,\ A = CA^*,$$
$$\xi = \tau\xi^*,\ x' = (b_0/\tau)(x')^*,\ x'' = (b_0/\tau^2)(x'')^*. \tag{1.9}$$

Here $\tau = b_0(\rho_0\lambda/p_0)^{1/2}$; $C = \pi\lambda p_0 b_0^2$; $p_0 = p(0)$; ξ is characteristic time for existence of pressure p; dimensionless variables are labeled with an asterisk. Below in numerical calculations, $p_0 = \rho_{EX}D^2/(k + 1)$ is Chapman–Jouguet pressure, ρ_{EX} and D are density of the explosive and detonation velocity, k is an index of polytropy for the detonation products. As a result of substitution for dimensionless variables, we obtain

$$x'' = \frac{(x')^2(1 - R - K) + S - p}{xR}; \tag{1.10}$$

$$-\sigma_r = (x')^2 - \rho_p[xx'' + (x')^2]\ln\frac{r}{x} + \frac{\rho_p(x')^2}{2}\left(1 - \frac{x^2}{r^2}\right) + \sigma_p \ln\frac{r}{x}; \tag{1.11}$$

$$A = -2\int_1^x pxdx,\ E_c = x^2(x')^2 R,\ E_1 = -\int_1^x x(x')^2 dx, \tag{1.12}$$
$$E_2 = [\sigma_c(b^2 \ln b - a^2 \ln a + a_0^2 \ln a_0) + \sigma_p(\lambda x^2 \ln x - b^2 \ln b)]/\lambda,$$
$$R(x) = \rho_c \ln\frac{a}{b} + \rho_p \ln\frac{b}{x} \geqslant 0,\ S(x) = \sigma_c \ln\frac{a}{b} + \sigma_p \ln\frac{b}{x} \geqslant 0,$$
$$K(x) = \frac{\lambda x^2}{2}\left[\rho_c\left(\frac{1}{a^2} - \frac{1}{b^2}\right) + \rho_p\left(\frac{1}{b^2} - \frac{1}{x^2}\right)\right] \leqslant 0,$$
$$a = (a_0^2 - \lambda + \lambda x^2)^{1/2},\ b = (1 - \lambda + \lambda x^2)^{1/2},\ \lambda = 1 - 1/\rho_p,$$

with $t < 0$, $p = 0$; with $t \geq 0$, $p = p(t) \geq 0$, $p(0) = 1$.

Initial conditions are $x(0) = 1$, $x'(0) = 0$, and p(t) is a monotonic fucntion falling normally with time. In (1.10)-(1.12) and subsequently in equations all of the values are dimensionless, and for simplification their asterisks are omitted. Reverse transfer to dimensionless variables may be carried out by Eq. (1.9) with previously restored asterisks for r, x, x', x'', t, p, A, E_i, ρ_i, ξ, σ_i, a_0.

By solving Eq. (1.10) it is possible to obtain relationship x(t), and by replacing it in (1.11) to find $\sigma_r(r, t)$. It can be seen from (1.10) that in the initial stage of compaction $x'' < 0$, since it is assumed that with small t, $p \approx 1 < S$, in other words, the shell does not deform. The SWF moves with acceleration towards the container axis, and there is acceleration of the container wall by external pressure created normally by detonation products. While the SW has passed a short distance from the inner wall of the container and the mass of compacted powder is small compared with the mass of the shell, for $x \approx 1$ with an accuracy to first-order terms for $\varepsilon = 1 - x \ll 1$, we have

$$x'' = \frac{(x')^2\,[1 - \rho_c\delta_c(1-\lambda) - \varepsilon(1 - \lambda + 2\lambda\rho_c\delta_c)] + \sigma_c\delta_c + \varepsilon(1-\lambda)\sigma_p - p}{\rho_c\delta_c + \varepsilon(1 - \rho_c\delta_c)}. \tag{1.13}$$

In deriving (1.13) use was made of the condition of container wall thickness smallness $\delta_c = a_0 - 1 \ll 1$. For quite small t, when $p \approx 1$ and $(x')^2$ is small, we obtain, by ignoring terms with $(x')^2$ and ε,

$$x' \approx \frac{\sigma_c\delta_c - 1}{\rho_c\delta_c}t,\ \varepsilon \approx \frac{1 - \sigma_c\delta_c}{2\rho_c\delta_c}t^2. \tag{1.14}$$

In this stage SW movement depends, apart from the applied pressure, mainly on shell mass and to a lesser extent on its strength properties, since in practice $\sigma_c\delta_c \ll 1$. Subsequently as pressure drops and ε increases, the strength properties and porosity of the contents of the container start to play a role. In the general case SW behavior depends on applied pressure and five dimensionless parameters: ρ_c, ρ_p, σ_c, σ_p, and a_0.

The asymptotic behavior of the solution of Eq. (1.10) as $x \to 0$ is found by means of substituting $y = (x')^2$, and p is considered as a function of x. Thus the equation is reduced to a first-order equation

$$\frac{dy}{dx} + My = N,\ M = -2\frac{1 - R - K}{xR},\ N = 2\frac{S - p}{xR}. \tag{1.15}$$

The solution of (1.15) is expressed in quadratures

$$y = g(x)\exp\left(-\int_1^x M(z)\,dz\right),\ g(x) = \int_1^x \exp\left(\int_1^z M(n)\,dn\right)N(z)\,dz. \tag{1.16}$$

As $x \to 0$, $y \to \infty$ like $x^{-2}(\ln x)^{\lambda-2}$, on condition that $g(x) > g_0 > 0$. If with $x \to 0$, $g(x) \to 0$ is not slower than $x^2(\ln x)^{2-\lambda}$, then y (SW amplitude) remains a finite value although it may also increase towards the container axis. Solution with $g(x) < 0$ corresponds to the case when the SW fades to zero without reaching the container axis. Thus, asymptotic behavior $y = (x')^2$ with an unlimited increase in SW does not depend on container parameters, and it appears to be the same as with absence of a shell. From asymptotics $(x')^2$ and expression (1.12) for E_c, it follows that $E_c \to 0$ with $x \to 0$.

2. In order to model container loading by an explosive substance in numerical calculations ξ is prescribed so that pressure pulse I (integral of pressure with respect to time) has the same value as in the case when p(t) is created by glancing detonation of an explosive layer. In calculations three different rules were used for the pressure with respect to time, i.e., exponential, linear, and power: $p = \exp(-t/\xi)$, $p = 1 - t/2\xi$, $p = (1 - \pi^2 t^2/16\xi^2)^{1/2}$

For all three relationships $I = \xi$ has one and the same value. The detonation pressure pulse, as also in [10], is determined from the relationship for rotation angle β of a plate thrown with glancing detonation and having a mass identical with that of the shell. An expression is used describing experimental results with an accuracy of 20%:

$$\beta = \frac{l}{l+2}\sqrt{\frac{3}{k^2 - 1}}, \tag{2.1}$$

where ℓ is ratio of explosive mass to that of the plate; k = 2.5 for ammonite and k = 2.8 for hexogen [11]. With small β ($\sin\beta \approx \beta$), the pulse received by the plate is proportional to angle β, detonation velocity, and mass of the plate. By equating I to the pulse of the plate per unit area, we obtain ($\delta_c \ll 1$) an expression

$$\xi = \frac{\rho_c\delta_c l}{l+2}\sqrt{\frac{3}{\lambda(k-1)\rho_{EX}}}. \tag{2.2}$$

Here Eq. (2.1) occurring for the planar case is used. With throwing of a cylindrical shell by an outer explosive layer, the values of the rotation angle appear less by 20-30% [12], which may be considered as corresponding to a reduction in ξ. In deriving (2.2) it was assumed that presence of powder does not markedly affect velocity acquired by the shell during acceleration. The effect is absent at a time when the mass of the powder compacted at instant of time $t = \xi$ appears to be much less than the mass of the shell ($\rho_c\delta_c \ll \varepsilon(\xi)$). If these values are comparable, then shell acceleration will occur more slowly, as though its mass increased. For $\rho_c\delta_c \gg \varepsilon(\xi)$, expression (2.2) is inapplicable.

Numerical solution of Eq. (1.10) was carried out by the fourth-order Runge–Kutta method, with values of parameters typical for practical applications. Results of numerical calcula-

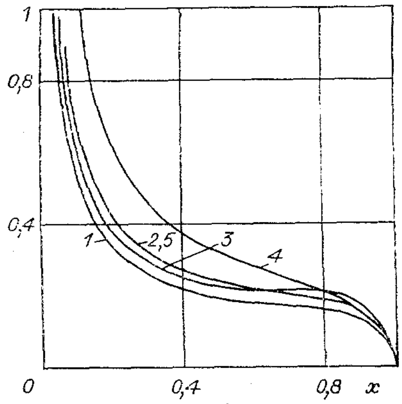

Fig. 1

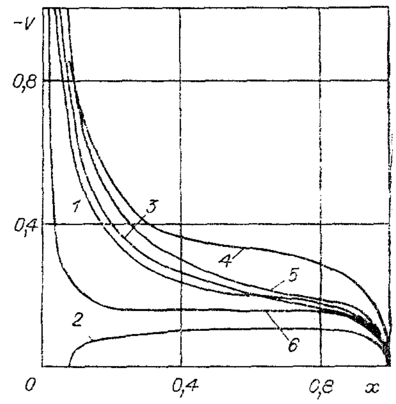

Fig. 2

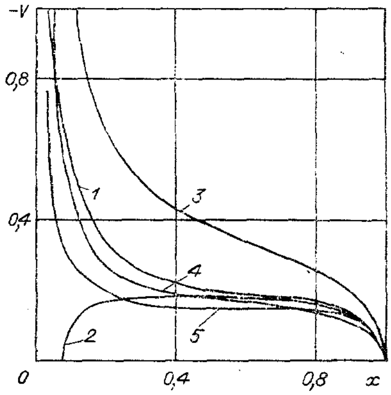

Fig. 3

tions are presented in Figs. 1-3 on dimensionless coordinates V—x, where $V = x'[\rho_{EX}/\lambda(k + 1]^{1/2}$ is the ratio of SW velocity to detonation velocity. Curve 1 in these diagrams was calculated with $a_0 = 1.33$, $\lambda = 0.666$, $\rho_c = 9.0$, $\rho_p = 3.0$, $\sigma_c = 0.203$, $\sigma_p = 0.00813$, $\xi = 0.776$, $\rho_{EX} = 1.10$, and an exponential relationship was used for p(t). This may correspond to the following values of physical quantities: container with inner and outer radii of 20 and 15 mm, a density of 9 g/cm³, dynamic yield strength of 0.5 GPa, powder with initial and final densities of 1.0 and 3.0 g/cm³, dynamic yield strength 0.02 GPa, layer of explosive (12 mm) with a density of 1.1 g/cm³ with a detonation velocity of 2.8 km/sec, k = 2.5. Curves 2 and 3 in Fig. 1 were calculated for linear and power relationships of p(t) with the same values of parameters. The solution appeared to depend weakly on the nature of pressure drop, and therefore further calculations were only carried out for the exponential relationship.

Each variant corresponded to a change in a potentially lower number of physical values used in calculating curve 1. Curve 4 in Fig. 1 relates to a change in explosive thickness to 18 mm and a detonation velocity up to 4.0 km/sec (with changing parameters $\sigma_c = 0.0996$, $\sigma_p = 0.00398$, $\xi = 1.32$), 5 to the original (12 mm) layer of explosive with k = 2.8 and detonation velocity 4.2 km/sec ($\sigma_c = 0.0978$, $\sigma_p = 0.00391$, $\xi = 0.708$).

Presented in Fig. 2 are the results of calculations with variation of container parameters. Line 2 corresponds to an increase in outer radius to 25 mm ($a_0 = 1.67$, $\xi = 0.807$), 3 to a reduction in container inner and outer radii to 15 and 10 mm ($a_0 = 1.50$, $\xi = 1.23$), 4 to a reduction in container density to 3 g/cm³ ($\rho_c = 3.0$, $\xi = 0.571$), 5 and 6 to a change in dynamic container strength to 0 and 1.0 GPa ($\sigma_c = 0$ and 0.406).

Given in Fig. 3 are calculated results obtained with varying material parameters. Curves 2-4 relate to an increase in dynamic yield strength to 1.0 GPa ($\sigma_p = 0.406$), initial density by a factor of two as a result of prior compaction ($\lambda = 0.333$, $\rho_c = 4.50$, $\rho_p = 1.50$), final density of the material to 4 g/cm³ ($\lambda = 0.750$, $\rho_p = 4.0$), and 5 to another material with initial and final densities of 3 and 9 g/cm³ with the same degree of prior compaction ($\rho_c = 3.0$). These calculations give an idea about the effect of change in different physical parameters of the experiment on SW behavior in the powder.

Numerical calculations showed the following. In many practically important cases, shell acceleration ceases when the SW has passed a distance corresponding to 5-15% of the container inner radius. The effect of powder strength properties on SW velocity is relatively weak. Curve 2 in Fig. 3, with attenuation of the SW to zero, was obtained with a dynamic yield strength (1.0 GPa) which can really hardly be achieved with an SW amplitude less than 0.2 GPa obtained from the calculation.

In a quite broad range of change in physical parameters, there exist solutions in which SW velocity changes weakly up to $x \le 0.1$. Subsequently there is a sharp reinforcement or weakening of the SW. Solutions with $x' \approx$ const in the region $1 > x_1 \ge x \ge x_2 > 0$, are possible when velocity changes in it are small, i.e., $|\Delta x'| \ll |x'|$. This condition with $\Delta x \sim 0.5$ is fulfilled when $|x''| \ll 2(x')^2 = 2y$ is valid. Whence, taking account of (1.10) we find that in this region SW velocity changes little if $2y \gg \left|\frac{y(1-R-K)+S-p}{xR}\right|$. When p is small in this region, then this inequality, due to the weak dependence on x of functions S, R, and K, easily provides a selection of parameters of the problem.

In order that $x' \approx$ const in region $1 > x_1 \ge x \ge 0$ taking account of the limitedness of p, fulfillment of an additional condition is necessary: $y(1 - R - K) + S - p \to 0$ with $x \to 0$. If a strictly constant SW velocity is required in the region $1 > x_1 \ge x \ge 0$, i.e., $x'' = 0$, then it is necessary that simultaneously in this region: $y(1 - R - K) + S - p = 0$ and $y = \sigma_p/\rho_c$.

As can be seen from Figs. 1-3 it is often the case that $|V| < 0.5$ for $x \gtrsim 0.1$. In the two-dimensional case with container loading by glancing detonation of explosive SW, slopes to the container axis of $\alpha < 30°$ correspond to these values of radial velocity. With such α the axial component of mass velocity for the specimen is small, and therefore in order to analyze the powder compaction process, it is possible to use a unidimensional model. If an unlimited increase in SW amplitude occurs, then starting with certain $x = x_*$, $|V| > 1$, SW velocity exceeds detonation velocity. Furthermore, simultaneous solution is known to be inapplicable for describing the actual compaction process. In the two-dimensional case, this behavior should relate to a smooth or sharp change in SWF slope to 90°, i.e., development of a "Mach disk." It is natural to assume that x_* corresponds to "Mach disk" radius. The SW then has the shape of a cone with a rounded tip or a truncated cone. This is the so-called "strong" regime, i.e., one of three loading regimes observed by experiment [13]. With a "weak" regime the SW attenuates to zero without arriving at the container axis. The intermediate regime is when the shock-wave surface is a right cone with a sharp tip, and x' = const up to the axis according to the model used would be difficult to accomplish in an experiment due to the limitations indicated above. Therefore, the intermediate region recorded in [13] is either not described within the framework of the given approach, or it may correspond to the case when the SW arrives with an approximately constant velocity almost up to the container axis, and then it accelerates (attenuates) sharply in a distance which is very small for reliable experimental recording.

A series of radial stress profiles in the powder $-\sigma_r(r, t)$ for different instants of time were calculated by Eq. (1.11) (Fig. 4, where series 1 relates to curve 1 in Figs. 1-3, 3 to curve 6 in Fig. 2).

We consider evolution of a radial stress profile within the container. The tangent to the slope of the curve $-\sigma_r(r, t)$ is

$$-\frac{\partial\sigma_r}{\partial r} = -U\frac{\rho_p}{r} + \frac{\rho_p y x^2}{r^3} + \frac{\sigma_p}{r}, \quad U = xx'' + y = [y(1-K) + S - p]/R. \tag{2.3}$$

It can be seen from (2.3) that while p is large and y is small, $U \le 0$, and then $-\partial\sigma_r/\partial r > 0$ for all $x \le r \le b$. Subsequently, when as a result of a reduction in p the U becomes quite large, on the curve $-\sigma_r(r, t)$ a maximum developes (Fig. 4). As a study of the sign of the second derivative shows, a minimum does not occur with any r and t. With further SW propagation the maximum may disappear. In the case of an unlimited increase in SW amplitude with $x \to 0$, as follows from asymptotic y, in the SWF $-\partial\sigma_r/\partial r > 0$, and a maximum always occurs as with absence of a shell [5].

For stresses at the boundary of the powder and container with r = b from (1.11), we have $-\sigma_r(b, t) = y - \rho_p U \ln\frac{b}{x} + \frac{\rho_p y}{2}\left(1 - \frac{x^2}{b^2}\right) + \sigma_p \ln\frac{b}{x}$. For instants of time when $\ln(b/x) \approx 1$, $-\sigma_r(b, t) \approx -\rho_p x x'' + y(1 - \rho_p/2) + \sigma_p$. Whence it can be seen that in particular with quite small

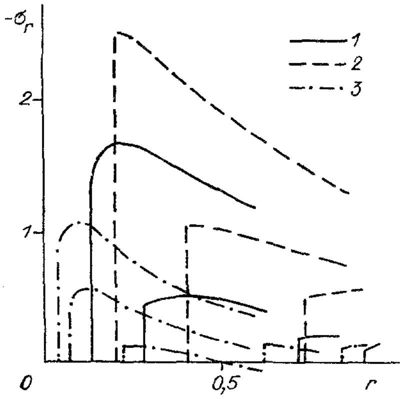

Fig. 4

σ_p and large ρ_p and with $x'' \geq 0$, $-\sigma_r(b, t)$ may be negative (Fig. 4), i.e., tensile stresses arise. The condition $x'' \geq 0$ may be realized with quite large dynamic container strength. Development of tensile stresses in practice may lead to occurrence of gaps between the powder and the container if the bond of the powder with the container appears to be insufficiently strong.

3. In order to obtain a qualitative idea about SW behavior in typical situations we carry out some estimates for a simple rule of p(t).

Let p(t) = 1 differ from zero for $0 \leq t \leq \xi$, ξ is quite small, so that $\varepsilon(\xi) \ll 1$ and $S(\xi) \approx \sigma_c\delta_c + \sigma_p\varepsilon(\xi)/\rho_p \ll 1$, i.e., time for the action of pressure is small and strength properties of the material up to $t = \xi$ are unimportant. The work of the forces of external pressure $A = 2\varepsilon(t)$ $(t \leqslant \xi)$. From (1.12) for small ε, it is possible to find that $E_2 \approx 2\sigma_c\delta_c\varepsilon + \sigma_p\varepsilon^2/\rho_p \ll A$. Over the time of pressure operation, there is shell acceleration and an increase in SW amlpitude y from zero to $y(\xi)$.

We consider the case of $\varepsilon(\xi) \ll \rho_c\delta_c$, and we use approximate solution (1.14), whence we find that $y(\xi) \approx 2\varepsilon(\xi)/\rho_c\delta_c \ll 1$, $\varepsilon(\xi) \approx \xi^2/2\rho_c\delta_c$. By using expression (1.12) for E_1, we find that at instant of time $t = \xi$ in plastic deformation in the SWF there will be energy expended $E_1(\xi) \approx [\varepsilon(\xi)]^2/\rho_c\delta_c$, i.e., a small part of $A(\xi) = 2\varepsilon(\xi)$. The work of plastic deformation behind the SWF (for the same time) $E_2(\xi)$ is also much less than $A(\xi)$. Therefore, at instant of time $t = \xi$ almost all of the work of the forces of external pressure appears to be stored in the form of kinetic energy of the shell, which in this model subsequently will be expanded in heating the powder and shell as a result of plastic deformation.

If an unlimited reinforcement of the SW does not occur, then total energy released in the SWF with convergence of the SW towards the container axis is

$$E_{1p} \leqslant -\int_1^0 y(\xi)\, x dx = \frac{A(\xi)}{2\rho_c\delta_c}; \tag{3.1}$$

with $\rho_c\delta_c \geqslant 1$ it consists of not more than half $A(\xi)$. If inequality $A(\xi) > A(\xi)/2\rho_c\delta_c + E_{2p}$, occurs, where E_{2p} is total energy consumed in deformation behind the SWF, then this means that SW amplitude should increase without limitation. (A limited increase in SW amplitude is possible according to (1.16) only with certain conditions on function g.) With $\rho_c\delta_c \ll 1$, inequality (3.1) means that after action of the external pressure ceases the SW amplitude should decrease rapidly with passage of the SW over distance $|\Delta x| \leqslant \rho_c\delta_c$.

In the second case, when $\rho_c\delta_c \ll \varepsilon(\xi)$, from (1.13) we obtain another approximate equation $x'' \approx [(x')^2 - 1]/[\rho_c\delta_c + \varepsilon(1 - \rho_c\delta_c)]$, whence

$$-x' \approx \sqrt{1 - \frac{\rho_c^2\delta_c^2}{(\rho_c\delta_c + \varepsilon)^2}}, \quad y(\xi) \approx 1, \quad \varepsilon \approx \rho_c\delta_c\left(1 - \sqrt{1 + \frac{t^2}{\rho_c^2\delta_c^2}}\right),$$

and from (1.12) it follows that $E_1(\xi) \leqslant \varepsilon(\xi) = A(\xi)/2$, i.e., in time ξ for plastic deformation in the SWF, it is possible to consume up to half of the total energy, and therefore after action of external pressure ceases, the characteristic distance in which the SW amplitude will decrease is $|\Delta x| \leqslant \varepsilon(\xi)$.

Thus, for short-term application of loads we obtain the following. With a shell of small mass ($\rho_c\delta_c \ll 1$), the SW amplitude decreases rapidly with distance. The distance in which the SW amplitude decreases to zero depends on σ_c and σ_p. With $\sigma_c = \sigma_p = 0$ there is always an unlimited reinforcement of the SW close to the axis [14].

For a massive shell ($\rho_c\delta_c \geqslant 1$) it is possible to accomplish a loading regime when the SW passes with approximately constant amplitude almost up to the container axis. The shell then reduces the maximum SW amplitude, and it promotes more uniform specimen compaction. Viewing the container as an explosive energy accumulator, which then consumes it in the work of plastic deformation, was formulated on the basis of experimental data in [2, 3, 12]. It is easy to see that in the model given this corresponds to the case of a massive container.

Similar energy estimates may also be carried out when a rigid rod is placed along the axis of a container with powder. Here after attaining the SW the rod may retain excess kinetic energy, and further calculation should be carried out taking account of the elastic properties of the materials. However, it is clear that in real specimens excess kinetic energy is converted to a significant degree into elastic deformation energy. This should lead to development of tensile stresses in the container and its contents during unloading, which will be greater, the greater the elastic deformation energy.

Thus, by means of a simple model, analysis has been carried out for the process of explosive powder compaction in a cylindrical container. It is governed mainly by the pulse of external pressure and five dimensionless parameters characterizing the powder and container. In the general case it is possible to separate three stages of SW convergence towards the container axis corresponding to three regions of the powder: outer, intermediate, and central. In the initial stage there is a rapid increase in SW parameters in the powder. The following stage is characterized by approximately constant SW velocity, and this may occur with a quite massive container. Here development of tensile stresses is possible within a container with powder. In the last stage, depending on values of dimensionless parameters, it is possible for there to be rapid attenuation or reinforcement of the SW leading to development of features close to the axis. Its reinforcement may be caused by convergence of the SW in cylindrical geometry and not connected with occurrence of an irregular reflection.

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